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An interesting consequence of the general principle of relativity

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Abstract. We show that Einstein's general theory of relativity, together with the assumption that the principle of relativity encompasses rotational motion, predicts that in a flat Friedmann-Lemaitre-Robertson-Walker (FLRW) universe model with dust and Lorentz Invariant Vacuum Energy (LIVE), the density parameter of vacuum energy must have the value $\Omega_{\Lambda 0} = 0.737$. The physical mechanism connecting the relativity of rotational motion with the energy density of dark energy is the inertial dragging effect. The predicted value is necessary in order to have perfect inertial dragging, which is required for rotational motion to be relative. If one accepts that due to the impossibility of defining motion for a single particle in an otherwise empty universe, the universe must be constructed so that all types of motion are relative, then this solves the so-called cosmological constant problem.

1 Introduction

Celebrating that it is now a hundred years since Einstein completed the general theory of relativity, we shall here investigate a consequence of this theory in the spirit of Einstein when he presented the theory in his great article that appeared in the spring 1916 [1].

In the present article we shall consider rotational motion and use the general theory of relativity to deduce a cosmic consequence of assuming that rotational motion is relative [2,3], meaning that an observer with a Fouceault pendulum on the North pole of the Earth, for example, may consider the Earth as at rest and the swinging plane of the pendulum as rotating together with the starry sky.

2 The significance of inertial dragging for the relativity of rotation

The first published paper on inertial dragging inside a rotating shell based on the general theory of relativity was published by H. Thirring [4] in 1918. He calculated the angular velocity of a ZAMO (Zero-Angular-Momentum Object) inside a shell with Schwarzschild radius R_S and radius r_0 rotating slowly with angular velocity, in the weak field approximation, and found the inertial dragging angular velocity,

$$\omega_d = \frac{8R_S}{3r_0}\omega. \tag{1}$$

This calculation does not, however, remove the difficulty with absolute rotation in an asymptotically empty Minkowski space. Both the angular velocity of the shell and that of the ZAMO are defined with respect to a system that is non-rotating in the far away region. There is nothing physically that determines this system since the far away region is empty. The absolute character of rotational motion associated with the asymptotically empty Minkowski spacetime, has appeared.

In 1966 D. R. Brill and J. M. Cohen [5] presented a calculation of the ZAMO angular velocity inside a rotating shell valid for arbitrarily strong gravitational fields, but still restricted to slow rotation, giving the expression

$$\omega_d = \frac{4R_S(2r_0 - R_S)}{(r_0 + R_S)(3r_0 - R_S)}\omega. \tag{2}$$

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For weak fields, *i.e.* for $r_0 \gg R_S$, this expression reduces to that of Thirring. But if the shell has a radius equal to its own Schwarzschild radius, $r_0 = R_S$, the expression above gives $\omega_d = \omega$. Then there is *perfect dragging*. In this case the inertial properties of space inside the shell no longer depend on the properties of the ZAMO at infinity, but are completely determined by the shell itself. Brill and Cohen further wrote that a shell of matter with radius equal to its Schwarzschild radius together with the space inside it can be taken as an idealized cosmological model, and proceeded: "Our result shows that in such a model there cannot be a rotation of the local inertial frame in the center relative to the large masses in the universe. In this sense our result explains why the "fixed stars" are indeed fixed in our inertial frame".

Their universe model is not particularly realistic. We do not live in Minkowski spacetime inside a cosmic shell. But that does not matter for the present argument. Their most important point was the introduction of the concept *perfect inertial dragging*. This will here be used to connect the assumption of the relativity of rotational motion with the value of the cosmological constant in an expanding universe model with Euclidean spatial geometry containing dust and LIVE (Lorentz-Invariant Vacuum Energy) with a constant energy density represented by the cosmological constant.

3 A cosmic consequence of assuming that rotational motion is relative

The distance that light and the effect of gravity have moved since the Big Bang is called the lookback distance, $R_0 = ct_0$, where t_0 is the age of the universe. *In order to have perfect inertial dragging in our universe, which is necessary in order that rotational motion shall be relative*, the Brill-Cohen condition implies that the Schwarzschild radius of the mass inside the lookback distance should be equal to the lookback distance.

At the present time the Schwarzschild radius of the cosmic mass inside the lookback distance is

$$R_S = \frac{2GM}{c^2} = \frac{8\pi G\rho_0}{3c^2} R_0^3, \tag{3}$$

where ρ_0 is the present density of all the mass and energy contained in the universe. Requiring that $R_S = R_0$ gives

$$t_0^2 = \frac{3}{8\pi G\rho_0}\,. \tag{4}$$

The present value of the critical density, corresponding to a universe with Euclidean spatial geometry, is

$$\rho_{cr0} = \frac{3H_0^2}{8\pi G}\,, \tag{5}$$

where H_0 is the present value of the Hubble parameter. Hence

$$t_0^2 = \frac{\rho_{cr0}}{\rho_0}\frac{1}{H_0^2}\,. \tag{6}$$

The present value of the density parameter and the Hubble age of the universe are defined by

$$\Omega_0 = \frac{\rho_0}{\rho_{cr0}}\,, \qquad t_H = \frac{1}{H_0}\,. \tag{7}$$

The assumption that rotational motion is relative thus leads to the relationship

$$t_0 = \frac{1}{\sqrt{\Omega_0}} t_H. \tag{8}$$

A large amount of different observations indicate that the density of the universe is very close to the critical density. Equation (8) shows that for a flat universe, *the assumption that rotational motion is relative, leads to the simple relationship*

$$t_0 = t_H, \tag{9}$$

i.e. that *the age of the universe is equal to its Hubble age*.

The standard model of the universe is a flat universe with dust and Lorentz Invariant Vacuum Energy, LIVE, with a constant density that may be represented by the cosmological constant, Λ. The present density parameter of the dust is Ω_{M0}, and of the LIVE is $\Omega_{\Lambda 0}$. The age of such a universe is given in terms of its Hubble age by [6]

$$t_0 = \frac{2}{3} t_H \frac{\operatorname{arctanh}(\sqrt{\Omega_{\Lambda 0}})}{\sqrt{\Omega_{\Lambda 0}}}\,. \tag{10}$$

Equations (9) and (10) lead to

$$\tanh\left(\frac{3}{2}\sqrt{\Omega_{\Lambda 0}}\right) = \sqrt{\Omega_{\Lambda 0}}. \tag{11}$$

The positive, real solution of this equation gives the following present value of the density parameter of dark energy, $\Omega_{\Lambda 0} = 0.737$. This is the prediction of the general theory of relativity together with the assumption that rotational motion is relative in our universe and that the universe has critical density and contains dust and LIVE.

4 Conclusion

The prediction that $\Omega_{\Lambda 0} = 0.737$ might of course be falsified by observations. It is remarkable that the WMAP and Planck measurements have given a present value of the density parameter of dark energy which is 0.73 ± 0.03 [7] in agreement with the predicted value.

This shows that relativity of rotation may be a physical reality in our universe, namely, this implies that the fact that the swinging plane at the North Pole rotates together with the stars, is no coincidence. It shows that the rotation of the plane is a gravitational effect from the cosmic masses upon the pendulum. The rotation of its swinging plane is due to perfect dragging from the rotating cosmic mass.

This means that every gyroscope is also acted upon by the inertial dragging effect due to the cosmic masses. The cosmic perfect inertial effect lines up the axis of a gyroscope so that it keeps on pointing at a fixed star even as the night sky rotates around it. This was used as a reference in the Lageos I and II experiments and the gravity probe B experiment that confirmed the existence of inertial dragging. The agreement of the predicted present value of the density parameter of dark energy and observations, confirms the existence of perfect cosmic dragging and implies that rotational motion is relative in our universe.

The density of vacuum energy is 120 orders of magnitude less that the quantum-mechanical cut off value, which is the Planck energy density. This is considered to be the quantum mechanical prediction for the energy density of the vacuum energy, leading to the largest known conflict between theory and observation.

We have shown that the general theory of relativity tells another story. It is not possible to define any type of motion for a particle that is alone in the universe. Motion can only be defined relative to other particles. One may therefore argue that the universe must be constructed so that if we have the correct theory, then this theory has to contain the principle of relativity for all types of motion. This was the way Einstein argued a hundred years ago inspired by Ernst Mach.

We have here investigated a consequence of assuming that the principle of relativity is valid for rotational motion in our universe according to Einstein's theory, and shown that if this is the case, and if the 3-space of the universe is Euclidean and dominated by dust and LIVE, then the universe must be constructed so that the present value of the density parameter of the vacuum energy is $\Omega_{\Lambda 0} = 0.737$.

Our calculation was motivated by a question about where the assumption that rotational motion is relative would lead us according to the general theory of relativity. It has turned out that the answer to this question has provided a solution of the cosmological constant problem. The solution is that the universe has to be constructed so that all types of motion are relative, since the opposite would imply that it should be possible to define the motion of a particle which is alone in the universe, which is not regarded as meaningful. This implies that LIVE with a constant density represented by the cosmological constant, contributes with an energy having 73.7% of the critical density of the universe.

References

1. A. Einstein, Ann. Phys. (Leipzig) **49**, 769 (1916).
2. Ø. Grøn, Nuovo Cimento B **7**, 861 (2010).
3. Ø. Grøn, Phys. Scr. **87**, 035004 (2012).
4. H. Thirring, Phys. Z **19**, 33 (1918).
5. D.R. Brill, J.M. Cohen, Phys. Rev. **143**, 1011 (1966).
6. Ø. Grøn, Eur. J. Phys. **23**, 135 (2002).
7. O. Lahav, A.R. Liddle, *The cosmological parameters* (Updated September 2011) available on-line at the following address: `http://pdg.lbl.gov/2011/reviews/rpp2011-rev-cosmological-parameters.pdf`.