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Minority persistence in agent based model using information and emotional arousal as control variables*

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Abstract. We present detailed analysis of the behavior of an agent based model of opinion formation, using a discrete variant of cusp catastrophe behavior of single agents. The agent opinion about a particular issue is determined by its information about the issue and its emotional arousal. It is possible that for agitated agents the same information would lead to different opinions. This results in a nontrivial individual opinion dynamics. The agents communicate via messages, which allows direct application of the model to ICT based communities. We study the dependence of the composition of an agent society on the range of interactions and the rate of emotional arousal. Despite the minimal number of adjustable parameters, the model reproduces several phenomena observed in real societies, for example nearly perfectly balanced results of some highly contested elections or the fact that minorities seldom perceive themselves to be a minority.

1 Introduction

Studies of opinion changes in societies are part of the core of topics of sociophysics. One of the reasons is the importance of understanding of changes in public attitudes versus specific issues or policies. The other reason is the natural way in which agent based descriptions of human behavior may be mapped into well understood models from statistical physics. There exists a wide variety of such approaches. Among the most popular, one can mention the voter model [1–4], the Sznajd model [5–12], the bounded confidence model [13–18], the Hegelsmann-Krause model [19], the social impact modef of Nowak-Latané [20,21] and its further modifications including the role of leaders [22–25].

Thanks to the application of ideas of statistical physics these works have pointed out several general properties of social behavior and have reproduced many specific aspects observed in real situations. However, in addition to these strong points, the sociophysics opinion models have also some weaknesses, limiting their application in some cases. Among them one may consider: focus on consensus (present especially in the early works, where the models used the analogies with magnetic systems); difficulties in describing the individual agent opinion dynamics (some models use psychologically implausible descriptions of individual agent reactions, for example assuming an automatic convergence of opinions resulting from a contact between two agents); artificial measures used to simulate discontent (solved sometimes via addition of ad hoc mechanisms simulating discontent, such as the introduction of contrarians into the existing models [26,27]) and finally the difficulties in connecting simulations and reality (for example mapping the Monte Carlo simulation steps into the "real world" time passage). Galam [28] has provided a general review of many two-state opinion models mentioned above, indicating the common origin of the opinion changes due to a specific form of a local majority influence. He also discusses weak points of these simulations, e.g. large differences in the times required to reach stability or the need to introduce agents with differing characteristics, especially with respect to the conviction and inflexibility of opinions. The model presented in this work treats all agents as identical, but introduces, in addition to opinion, an emotional state.

Our goal is to present a model that would be based more closely on psychological understanding of human behavior, yet be still simple enough to allow effective treatment in simulated environments and mapping of the system variables into generally understood social and psychological concepts. Part of the motivation comes from extended studies of the behavior of the users of Internet fora, where we have observed strong correlations between the expressed opinions and emotions of the participants [29–31]. Notably, we have observed that when the emotional arousal of the participants is high, their capacity to change opinions is negligibly small. This observation has led us to propose an approach in which the individual opinion about a specific issue would be influenced by a combination of the information related to the matter and the emotional state of the person. The other part of the motivation comes from the agent based models of emotional behavior [32,33]. It is our belief that coupling

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the description of opinion and emotional changes can be beneficial for the understanding of both phenomena.

2 Model description

The current paper is based on the model introduced in reference [34]. The goals of the model are very simple: to account for the observed individual psychological characteristics within a simple framework and to combine such "microscopic" description with a flexible communication mechanism allowing to use the model in different social contexts.

2.1 Psychological background for the model: catastrophe theory

The proposed solution is based on a simplification of the catastrophe theory of behavior, which has been introduced more than 30 years ago [35], and had been used (and criticized) in analyses of human behavior (for a review see [36]). In the context of individual opinion changes, the most popular approach is based on the cusp catastrophe, which allowed to intuitively explain a hysteresis behavior observed in many situations [37–45]. This approach identifies two control variables that determine the resulting opinion. One of these variables is the normal factor (for example information available on the issue and supporting one of the alternatives). The other variable, splitting factor (in our case taken to be the emotional arousal), leads to the appearance of a region of the control parameters where two values of the opinion are allowed for the same information and emotional level (see Fig. 1). The cusp catastrophe model allows to describe situations where the same amount of information that would lead to an opinion change for low emotional state would leave the agent's opinion unchanged for high value of the emotional arousal.

2.2 Discrete state model

While the catastrophe theory has been used in description of the individual behavior, it is not well suited for large scale opinion simulations. This is because the continuous nature of the control variables makes it very difficult to correctly map the model and psychological observations and then to assign these values to computer based agent societies – there is simply too much variability in the starting conditions and system evolution. Such excessive freedom of choice, without capacity to compare with real societies, would diminish the predictive capacities of the models. For this reason we have proposed [34] a discrete version of the approach, in which the continuous folded cusp surface is replaced by just seven states corresponding to two values of the emotion level (splitting factor): C(alm) and A(gitated) and three values of the information about the issue in question, where we used +1 (P), −1 (M) and 0 scale (Fig. 1). All the states may be fully described via a three letter acronym. The first letter corresponds to the emotional state, the second to the direction

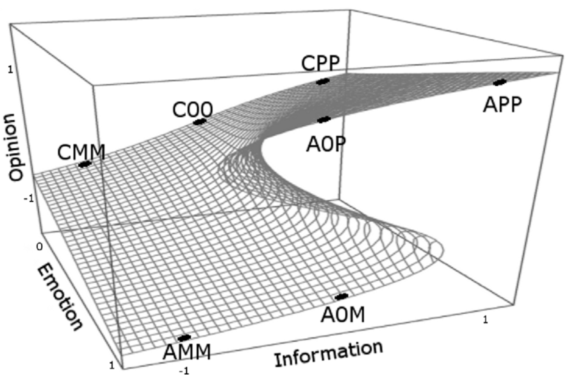

Fig. 1. Schematic representation of the seven states of the agents, depending on emotion (E) and information (I) state, showing relationship of the current discrete model to the continuous cusp catastrophe. The two control variables are information (normal factor) and emotion (splitting factor). The states are denoted by three letter acronyms: the first letter describes the emotional state C (alm) and A (gitated), the second the valence of the information related to the subject P (lus 1), M (inus −1) or 0, while the third letter denotes the final opinion of the agent, using the same convention P (lus 1), M (inus 1) or 0. Thus, for example, CPP denotes a calm agent who, based on positive information has positive opinion. In the agitated state ($E = 1$) the agent may support one of the two conflicting opinions for the same value of emotion and information (A0M and A0P). Instead of continuous paths over the cusp surface, the development of an opinion is described through jumps between the states.

the information held by the agent points to, and the third letter to the resulting opinion held by the agent. For calm agents the opinion is always in line with the information available to the agent, so we have three states: CPP, C00 and CMM. In the agitated state, the agent may base its opinion on the available information (the APP and AMM states). The most important aspect of the discrete model is that there are two states (A0P and A0M) when the information available to an agent is inconclusive, but the agent still has a definite opinion. This case corresponds to the folded part of the surface in the continuous cusp catastrophe. We note that there is no A00 state – all the agitated agents are assumed to have a definite opinion, even if they have no supporting information. This corresponds to the unavailability of the "reversed" part of the cusp surface in the continuous model.

2.3 Individual agent dynamics

The use of discrete states allows a simple description of behavior of an individual agent resulting from contact with another member of the society or with an external information/emotion source. Throughout the paper we are using an approach based on communication via discrete messages. Such message – originating from another agent or from the press, TV, the Internet or other media – would be described by exactly the same set of variables: emotional arousal level, information and opinion. In the case of messages sent by an agent, the characteristics of the message are assumed to equal those of the authoring agent.

Table 1. Matrix of states of agents resulting from a single message sent by the "Sender" and received by the "Recipient" in given state. Each cell is the final state of the recipient. Only the changed recipient states are noted. In the case of CPP message received by a CMM agent (or vice versa) there are two possible outcomes: with probability p_a, the recipient of the contrary message may get agitated, changing its emotional state from C(alm) to A(gitated), without changing the information nor the opinion, CMM → AMM. With probability $1 - p_a$ a calm contrarian message may convince the calm recipient to change its information and therefore, opinion, resulting in transition CMM → C00.

Recipient	Message content (state of the Sender)						
	CMM	C00	CPP	AMM	A0M	A0P	APP
CMM			AMM (p_a) C00 ($1 - p_a$)			AMM	A0M
C00	CMM		CPP				
CPP	APP (p_a) C00 ($1 - p_a$)			A0P	APP		
AMM	CMM		A0M				A0M
A0M	C00						
A0P			C00				
APP	A0P		CPP	A0P			

Upon receiving a message, its recipient may, in some cases, change its state. This is described by a transition matrix (Tab. 1). In most cases, the message does not lead to any change. For example, a CPP agent receiving a CPP message would simply feel "reinforced" in its state. C00 messages, carrying no information nor emotion, leave the recipients unchanged. There are, however, situations where the message would lead to a change of the state. For example, an agitated agent receiving a calm message supporting its opinion may calm down. Or the opposite: a calm agent confronted with an agitated message supporting the opposing opinion would get angry and change the state to an agitated one. An agitated agent receiving the contrary message would at most nullify its information about the issue, but would not change its opinion. Thus, in our model, any agent, in the emotionally agitated state, plays the role of "inflexible agent" as introduced by Galam [46] or "zealot" [47,48]. Instead of introducing special classes of agents (inflexibles, contrarians [26,27,49], independents and conformists [50]), we postulate a differing opinion dynamics of any agent, depending on the emotional state, which may change itself as the result of inter-agent interactions. We note here that the transition rules (Tab. 1), with just two exceptions, are fully deterministic. They correspond to intuitively understood reactions from everyday life.

The two exceptions from strict determinism of individual dynamics are reactions of calm agents of definite opinion (CPP and CMM) to calm messages supporting the opposite views (CMM and CPP, respectively). In this case we assume that with "arousal" probability p_a the agent may react to the message by rejecting its content and becoming agitated. In a sense, the p_a measures the agent's irritability or irrationality. The other, "rational" outcome is the change of the state of the agent to C00 (with probability $1 - p_a$). This corresponds to the change of the information and opinion due to calm, cognitive acceptance of contrary information. The arousal probability p_a, measuring the agent irritability and irrationality is one of the two variable parameters in the current study.

2.4 Message based communications

As already noted the model uses messages based communication process. There are two reasons for this choice. First, it allows a direct application of the model to many social systems, notably to most forms of ICT communications (e-mails, discussions, tweets, blog posts...). The second reason is that such approach would allow to map the simulation time to real time via the number of exchanged messages (which can be recorded separately from their content). Throughout the paper we will use as time measure the average number of messages per agent. The simple form of the model on which we focus assumes that all agents communicate randomly, with the same probability – an assumption that might not be valid for specific social systems.

2.5 Starting conditions

For most agent based simulations the results depend on the assumed initial conditions. The topic is often neglected in the discussions of results. The freedom of choice of assignment of the initial opinions (and, in our case, emotions) may lead to huge numbers of differing results, where the determination if a specific effect is the result of general dynamics of the system or of the particular choice of starting conditions is very difficult. In turn, this stalls the application of the simulations to real world conditions. For example, many simulations choose as the starting configuration a random distribution of various characteristics of the agents within the social space. Such assumption is well justified in the case of the corresponding physical "spin-based" models (especially if the initial conditions correspond to high enough temperatures). But such randomness may be totally inappropriate in social situations.

To avoid this trap we have chosen, for all the simulations in this paper, a very simple starting configuration. We assume that the initial configuration is composed mostly (99%) of agents who do not have any information about the considered issue and who are in calm state

(C00). Only the remaining 1% of agents "know" something about the issue. Further we assume that they are divided asymmetrically (0.3333% are in the CPP state and 0.6667% in the CMM state). These initial "seeds" are distributed randomly within the population. These initial conditions have a direct counterpart in some specific social environments, namely when a new issue is introduced to a population that has not been considering it previously.

While this choice is entirely arbitrary, we hope it might represent at least these social situations where the opinion relates to a new issue, previously unconsidered by the community. However, to provide some understanding of the consequences of the choice of a particular starting condition setup, we have appended a more thorough discussion of the simulation results dependence on some characteristics of the initial conditions in the supplementary information.

3 Solutions for infinite range interactions

In reference [34], we have considered the evolution of the system in the simplest communication pattern, where any agent in a community may send messages to any other agent. This has allowed to study the behavior without considering any specific topology of the social network and to derive difference equations for the global variables (such as the relative ratios of agents in specific states). The computer based simulations starting from specific realizations of the communication process (random choices of the communicating agents) led to results very close to the solutions of these difference equations.

The present paper extends these calculations by including the effects of the arousal probability. The difference equations for the ratios of agents in specific states (normalized to the whole society size) are:

$$\begin{aligned}\Delta P_{CMM} &= P_{CMM}(P_{C00} - P_{CPP} \\ &\quad + P_{AMM} - P_{A0P} - P_{APP}) \quad (1)\end{aligned}$$

$$\begin{aligned}\Delta P_{C00} &= 2P_{CMM}P_{CPP}(1-p_a) \\ &\quad - P_{CMM}P_{C00} - P_{CPP}P_{C00} \\ &\quad + P_{CMM}P_{A0M} + P_{CPP}P_{A0P} \quad (2)\end{aligned}$$

$$\begin{aligned}\Delta P_{CPP} &= P_{CPP}(P_{C00} - P_{CMM} \\ &\quad + P_{APP} - P_{A0M} - P_{AMM}) \quad (3)\end{aligned}$$

$$\begin{aligned}\Delta P_{AMM} &= P_{CMM}P_{A0P} - P_{CMM}P_{AMM} \\ &\quad - P_{CPP}P_{AMM} - P_{AMM}P_{APP} \\ &\quad + p_a P_{CMM}P_{CPP} \quad (4)\end{aligned}$$

$$\begin{aligned}\Delta P_{APP} &= P_{CPP}P_{A0M} - P_{CMM}P_{APP} \\ &\quad - P_{CPP}P_{APP} - P_{AMM}P_{APP} \\ &\quad + p_a P_{CMM}P_{CPP} \quad (5)\end{aligned}$$

$$\begin{aligned}\Delta P_{A0M} &= P_{CMM}P_{APP} - P_{CMM}P_{A0M} \\ &\quad + P_{AMM}P_{APP} + P_{CPP}P_{AMM} \quad (6)\end{aligned}$$

$$\begin{aligned}\Delta P_{A0P} &= P_{CMM}P_{APP} - P_{CPP}P_{A0P} \\ &\quad + P_{AMM}P_{APP} + P_{CPP}P_{AMM}. \quad (7)\end{aligned}$$

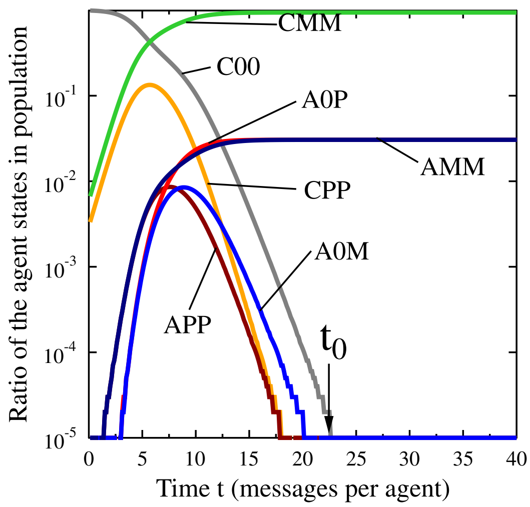

Fig. 2. Example of solution of the difference equations for any-to-any, infinite range communication mode. Note the logarithmic vertical scale. The system evolves to the final stable state (reached at t_0) via somewhat complex intermediate stage when all possible agent states are present; $p_a = 0.1$.

For the considered initial conditions with no agitated agents (assuming that only a small fraction of "seed" agents have predetermined opinions and that one side has initially somewhat larger number of supporters and for $p_a = 0$), the stable final condition is fully dominated by the majority CMM opinion. Within the same starting conditions, allowing the agents to become agitated upon CMM/CPP communication ($p_a > 0$) leads to final stable conditions with some fraction of the population in the CMM state and the rest divided equally between agitated agents in A0P and AMM states (Fig. 2). The agitated minority (A0P) exists despite their information about the issue is non-decisive (thanks to the combination of initial "positive" information value and the influence of the negative surrounding majority), because their emotional state fixes each individual into a denial state.

It is important to note that the infinite range of communication leads to the final outcome with only three types of agents (the only exception is when the initial conditions are perfectly balanced, but this is an unstable solution). The final number of the agitated agents depends on the ratio of the initial informed seeds in the neutral background and on the relation between CPP and CMM seeds.

4 Finite communication range simulations

While the any-to-any, infinite range interaction mode has the advantage of allowing the solutions using difference equations, we note that such communication network is relatively unusual in the real world. Even in environments where it would be possible technically (such as Internet communities), the real connection networks are characterized by well described local communities. For this reason we turn our attention to more "localized" interactions,

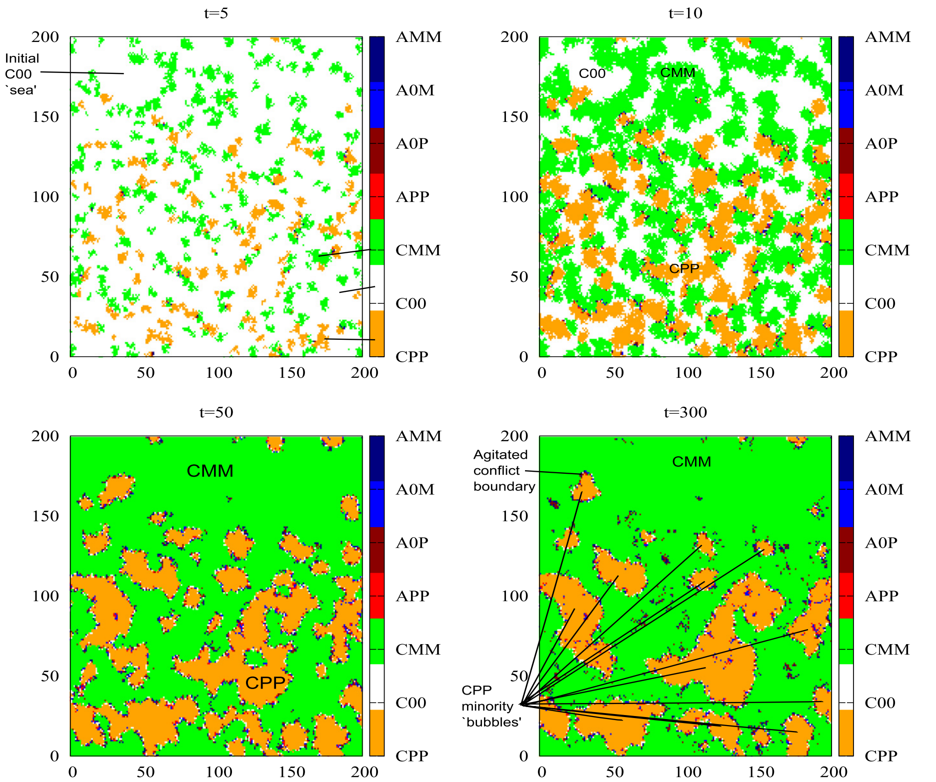

Fig. 3. Snapshots of the system evolution at four times (measured by the average number of messages per agent) ($t = 5, 10, 50$ and 300). Pastel colors indicate calm agents, darker colors show agitated ones. At $t = 5$ bubbles of opinion form around the initial "seeds". At $t = 10$ larger structures connecting agents of the same opinion begin to form and opposing agents start to meet. At later times the dynamics of the system is much slower, driven by local "conflicts" at the boundaries between the two large communities. For short range of communications these conflicts are confined to these boundaries; simulation results for $p_a = 0.5$, $N_C = 8$ (Moore neighborhood).

where the contacts between agents are limited by specific form of the social connectivity network.

Social communication networks may take various forms, differing in their topology and time distribution of messages. One of the variants found in many environments are scale free networks [51–54]. In this study, however, we decided to use a simple square-based planar topology with limited range interactions. The main reason was to allow intuitively accessible visualizations of the results of the simulations, providing insights into the driving processes. Additionally, the use of fixed geometry as the communication topology allowed to decrease the number of uncontrolled variables in the simulations (such as the differences in individual agent connectivity) and therefore to focus on the global effects of the microscopic dynamics. In general, the proposed opinion change model is independent of the social network topology and may be used in studies of scale free networks, including also time-dependent, evolving structures.

The geometry we have used is based on a 200×200 grid of agents, with periodic boundary conditions. The messages between the agents are limited to specific neighborhood of fixed size. The number of communication agents N_C is the second variable driving the behavior of the system. The smallest is the von Neumann neighborhood ($N_C = 4$), increasing to the Moore neighborhood ($N_C = 8$), Manhattan distance of 2 ($N_C = 12$), and then the increasing Chebyshev distances of 2, 3, 4, 5, 6... ($N_C = 24, 48, 80, 120, 168\ldots$). An agent may receive a message from any of the agents in such neighborhood with equal probability.

The evolution of the system may be roughly divided into three stages. In the first stage, when the major part of the society is in the calm, uninformed state C00, the initial "advocates" of one opinion or the other (CPP and CMM seed agents) convince the neighboring C00 agents to their view by supplying them with the information. During this stage "bubbles of opinion" form around the seeds ($t = 5$ panels in Figs. 3 and 4). Especially, in situations where the communication range is small, the initial evolution regions where the opinions are uniform. As the information spreads within the society (C00 agents turn into CPP or CMM) these bubbles begin to merge, forming more complex structures and start to bring the opposing CPP and CMM agents in direct contact (Figs. 3 and 4, $t = 10$ panels). For $p_a > 0$, at the edges of these structures, as the result of the difference of opinion, agitated agents begin to accumulate. For short range interactions these inflexible, agitated agents isolate the calm subcommunities which persist with little changes long into the simulation time ($t = 50$, $t = 300$ panels in Fig. 3). For simulations with long range interactions (e.g., spanning the Chebyshev distance of 7 or $N_C = 224$ neighbors, Fig. 4) the conflict is not limited to the boundaries between the CPP and CMM "territories". Agents may become aroused within the calm "bubble", through contact with relatively distant opponents. As a result, at late simulation times the calm minority subcommunity (in this case CPP) may disappear altogether, leaving only the agitated A0P opinion holders, surrounded by the CMM and AMM agents. This configuration is a stable, final stage of the system evolution.

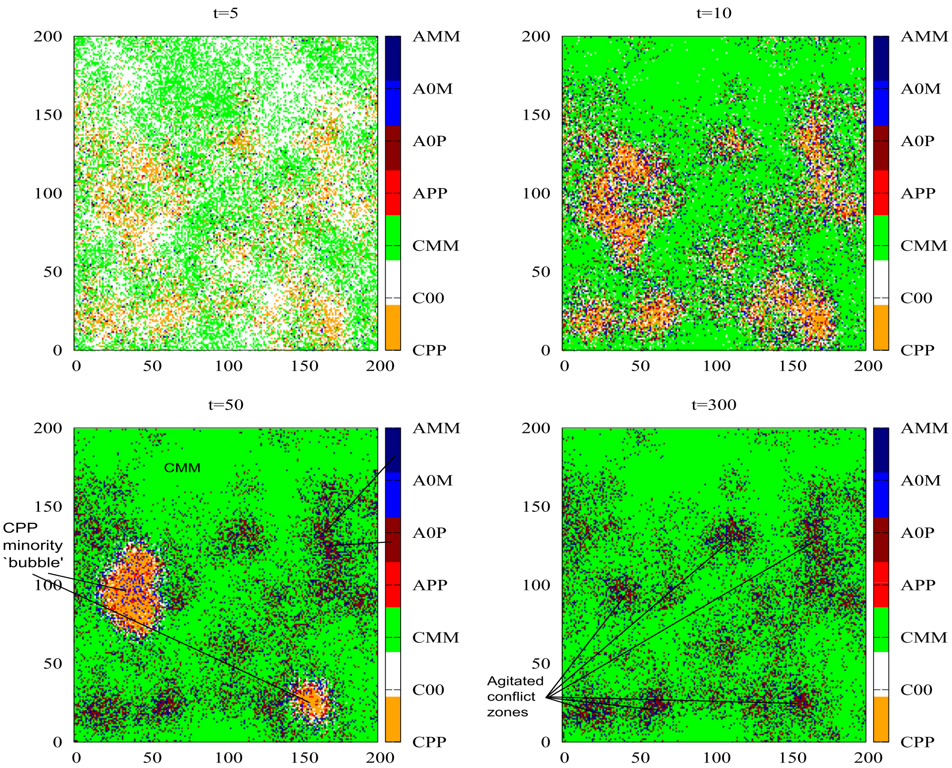

Fig. 4. Snapshots of the system evolution for long lange interactions (number of neighbors $N_C = 224$, Chebyshev distance of 7), other parameters as in Figure 3. The longer range of interactions "allows" the quarrels to "penetrate" into the communities of calm agents and increase the ratio of agitated agents. Eventually ($t = 300$ panel), the process leads to the disappearance of the calm minority communities, leaving only agitated minority "diehards" A0P and emotional agitators for the majority view AMM, localized in "conflict zones".

4.1 Evolution of system characteristics

The influence of the finite range of communications (as shown in the visualizations in Figs. 3 and 4), leads to modification of the long term behavior found in the case of infinite-range, any-to-any communications. The main change is the appearance of an intermediate stage of coexistence of two calm communities, separated by the "conflict zones" (Fig. 5). The longevity of this metastable stage (between t_1 and t_2, indicated in Fig. 5) depends on the communication range: for very local communications it may be very long indeed. Increasing the number of communicating agents N_C shortens the intermediate period, and leads faster to the final stable state, similar to the one found in the infinite communication range case.

The initial ($t < t_1$) behavior of the growth of the opinion holding calm agents (majority CMM and minority CPP) is similar to the logistics equation, as in the population dynamics. The modifications are important especially for the short communication distances, and result from the presence of contacts within the already "infected" agents of the same type. Increasing the communication range allows the initial process to be accurately described by the standard form of $(ax_0 \exp(t))/(a + x_0(\exp(t) - 1))$, with x_0 being the initial ratio of CPP or CMM agents and a corresponding to the value at t_1. The initial phase in all cases ends up rather quickly, t_1 is typically below 20 for very short range communications and well below 10 for longer interactions. This means that the initially uninformed majority of C00 agents gets convinced, on way or another, very fast, even for localized communications.

The second characteristic time, t_2, when the system reaches the stable state, shows much greater variability between the individual simulation runs for the same set of parameters and initial conditions. Running a large number of simulations we were able to determine the dependence of the average values of $\langle t_2 \rangle$ on N_C (Fig. 6). This is described, to a high accuracy with a power law relation with the exponent of about -1.1. The distribution of t_2 values in individual runs is also interesting. For short and medium range communication, from time to time, we have encountered simulations in which the intermediate phase was extended apparently indefinitely – up to simulation times longer than 10 average t_2 values. The origin of such extremely long intermediate states is due to the particular choice of the interaction geometry and may have little correspondence with social phenomena. Figure 7 presents the agent state distribution for one of such simulations at $t = 4000$. We note that in this specific case the calm minority has evolved to a horizontal stripe and the boundary between it and the calm majority is flat. With the interaction range smaller than the width of the strip, the agents at both sides of this boundary have the same probabilities of conversion – which leads to the system stability. Such stability is absent in the case of more rounded "bubbles", as shown in Figures 3 and 4, which are typical for most simulations.

4.2 Dependence on p_a and N_C

We turn now to the specific dependence of the global system properties (such as the ratios of agent states, average opinion and average emotional agitation ratio) on the two parameters of the model: p_a and N_C. We have run the simulations for different initial seed spatial distributions

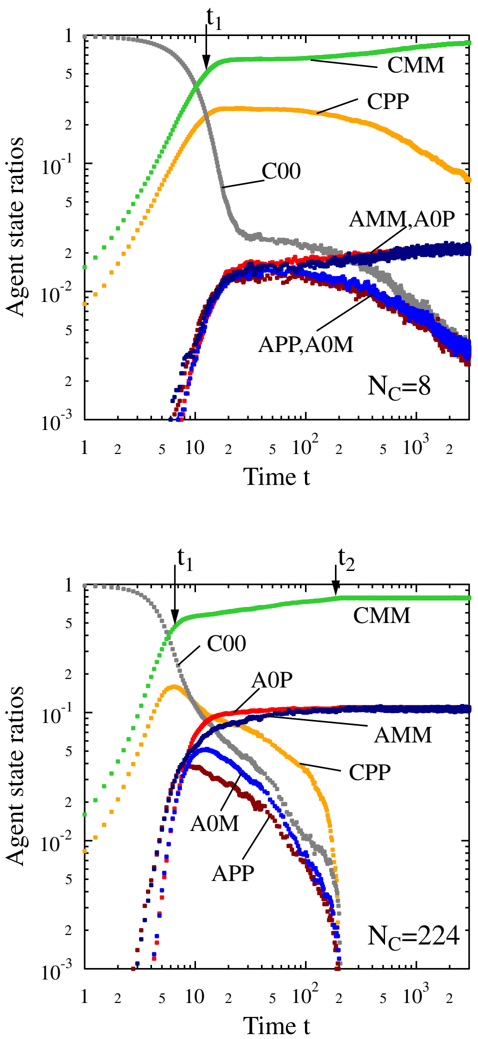

Fig. 5. Specific examples of the long term evolution (up to 3000 messages per agent) of agent states ratios for the parameters corresponding to Figures 3 (top) and 4 (bottom). Note the logarithmic scale on the time axis. The evolution may be divided generally into three stages: initial formation of opinionated “bubbles” in the generally uninformed society ($t < t_1$); the intermediate stage of conflict between the majority and minority ($t_1 < t < t_2$), when, depending on the communication range the calm majority overcomes the calm minority with differing speed, but the evolution is generally slow; and the final stable state with only the calm majority (CMM) and two approximately equal agitated “fighting” groups (A0P and AMM) remain ($t > t_2$). For the short range communications (upper panel), the end time of the intermediate stage t_2 is greater than the scale of the figure; $p_a = 0.5$.

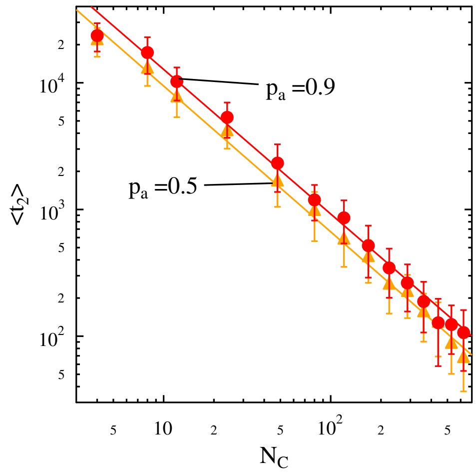

Fig. 6. Dependence of the average end time of the intermediate stage $\langle t_2 \rangle$ on the communication range (number of communicating agents N_C). Specific simulations lead to widely varying results, as indicated by the error bars, which are the standard deviation values for many simulations. Values used in simulations: $p_a = 0.5$ and $p_a = 0.9$, initial conditions as in other examples. The lines are power law fits with the exponent of -1.13 for $p_a = 0.5$ and -1.09 for $p_a = 0.9$, both very close to inverse proportionality between t_2 and N_C.

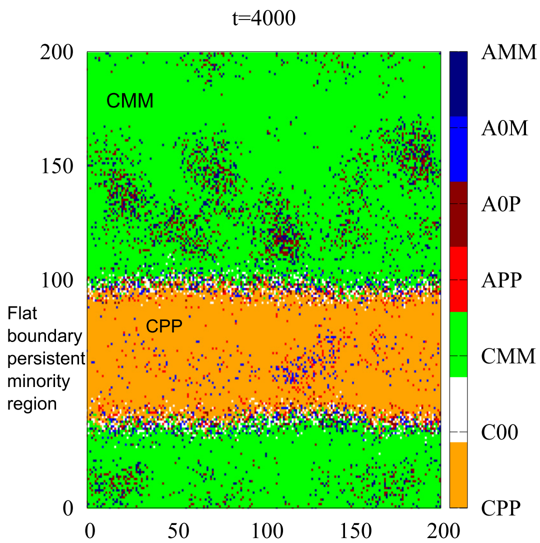

Fig. 7. Special configuration corresponding to an extremely long persistence of the intermediate state. The calm minority has evolved into a horizontal strip with flat boundary with the calm majority. Under such conditions, the global state remains stable due to the particular communication topology of the square lattice; $p_a = 0.5$, $N_C = 224$.

(keeping the initial composition constant), with the values of p_a of 0 (no agitation), 0.1, 0.5 and 0.9. We remind here that these three cases correspond to the arousal of emotion in a CPP/CMM encounter in 10%, 50% and 90% cases, respectively. We have run the simulations for increasing number of agents in the local communication neighborhood, starting from $N_C = 4$ up to $N_C = 624$. We have arbitrarily chosen $t = 300$ as the “observation time” to search for the effects of finite, “reasonable” number of messages on the system behavior.

As in the case of the t_2 values, there were significant differences between simulation runs using the same set of parameters. They resulted from spatial differences in the initial distribution of CPP and CMM seeds and from different histories of agent to agent communication. Figure 8 presents the global ratio of agents in each state as

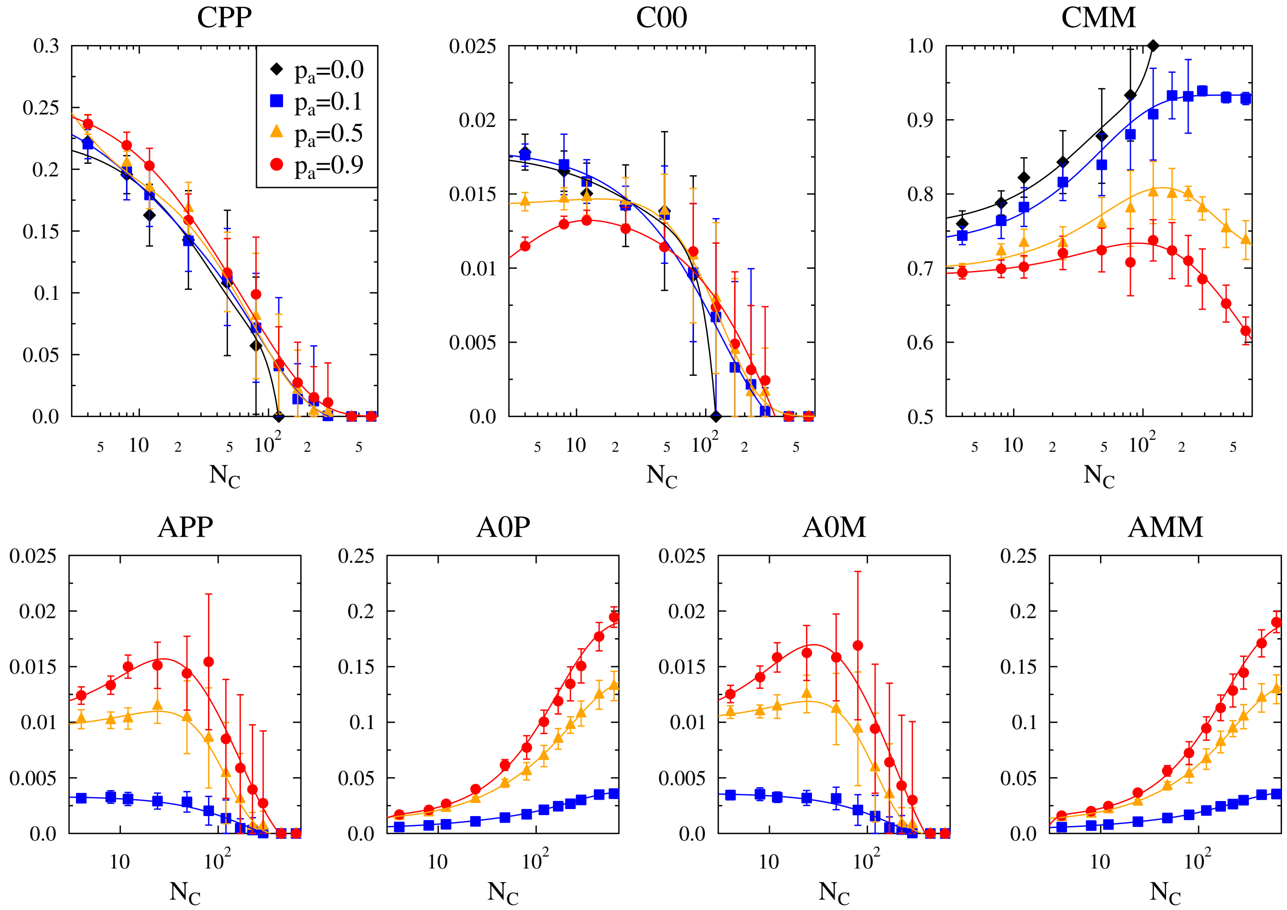

Fig. 8. Relative numbers of agents in each state at the time of $t = 300$ (300 messages per agent) as function of the communication range N_C, for different values of the arousal probability p_a. As the simulations start from mostly uninformed calm society, when the arousal probability $p_a = 0$ (black diamonds) there are no agitated agents. In this case, the minority vanishes if the communication range is greater than 120 neighbors. The whole system becomes uniform, calm majority CMM. In the cases where agitation is possible, the final number of CPP and C00 agents decreases more slowly with the increased interaction range, but also vanishes for $N_C > 250$. In such situations the final system is composed of the calm majority CMM and equal numbers of agitated agents of AMM and A0P. This is in complete agreement with the simulations for the infinite range of interactions (any-to-any agent). The lines are best fits of empirical dual-exponential functions to the average values of number of agents fin each state at $t = 300$ and serve to guide the eye.

functions of p_a and N_C at time $t = 300$. This time is, in all cases, much greater than the end of the expansion phase t_1. For some combination of the parameters it is lower than t_2, while for others (especially for large N_C) it is higher than t_2. The reason for large fluctuations of these specific agent state ratios at $t = 300$ (shown by the error bars) is that depending on the specific simulation run certain “bubbles” of minority agents may have survived or vanished within the studied time. These differences in final outcome are especially important for the intermediate values of $24 \leq N_C \leq 224$, which correspond to communications distances between 2 and 7 squares away from an agent. The lines in Figure 8 are best fit of the form $a + b_1 \exp(-N_C/n_1) + b_2 \exp(-N_C/n_2)$ and serve to guide the eye.

For the $p_a > 0$ cases we can divide the agents at the intermediate stage of simulation into three groups. First, there is a small number of neutral C00 agents, not greater than 2% of the population, decreasing with increasing N_C. The second group consists of the majority calm agents (CMM) and the associated pairs of quarrelling agitated ones (AMM and A0P). These pairs are “embedded” in the calm population and preserve their opinions due to high emotions despite being surrounded by majority of the opponents. The third group is a symmetrical one: calm minority (CPP) and associated with it the quarrelling A0M and APP agents. Obviously, increasing the p_a value increases the number of agitated agents.

Increasing the range of communication allows to communicate within the enclaves of calm agents. As shown in Figure 4, this leads to the eventual disappearance of the calm minority and the associated agitated agents in the A0M and APP states. On the other hand, the size of the calm majority depends non monotonically on the communication range. At first, increased N_C leads to an increase of the number of CMM agents (mostly at the cost of CPP

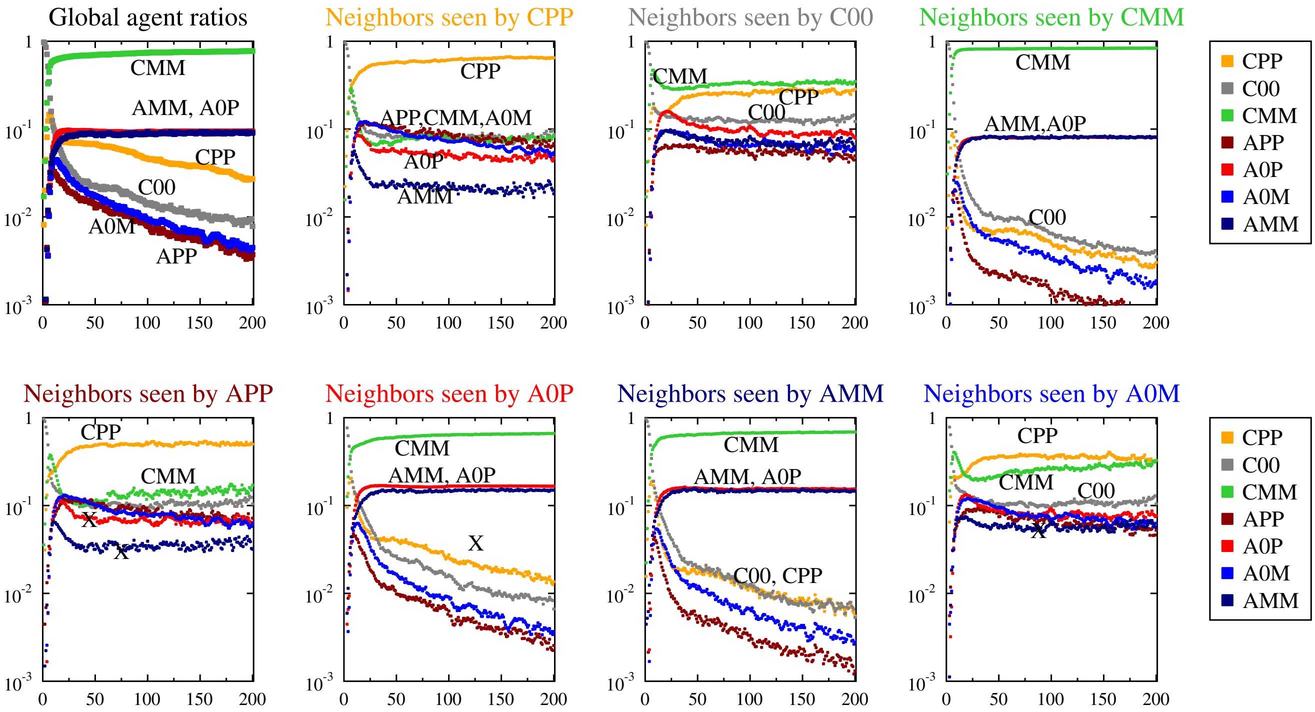

Fig. 9. Comparison of the evolution of numbers of agents in each state (CPP, C00, CMM, APP, etc.) as function of simulation time, measured by the average number of messages per agent. The simulation parameters as in Figure 4 ($p_a = 0.5$, $N_C = 224$). Top left panel shows the actual ratios of agent states for the whole system. Note different behavior for the initial phase of converting the uninformed (up to $t = 15$) and the later phase of conflict between supporters of two views. For the relatively large value of communication range the calm minority vanishes rather fast. In simulations with very short range communications, the system "freezes" with almost constant values after the initial phase. The remaining panels show the average ratios observed by agents of each type within the range of communication, top level for the calm agents, bottom one for the agitated ones. It is clear that certain types of agents (for example the calm minority CPP) "perceive" their world in opposite to the actual situation: their environment consists mostly of their own type, so they may "believe" that they are the majority!

and C00 ones). However, for $N_C > 250$ this growth stops and, for larger p_a values, even reverses. At the same time the numbers of the "majority fighters" (AMM) and the "minority diehards" (A0P) increases monotonically with increasing N_C.

We note here that as the agitated agents' distributions are matched in two balanced groups (with paired numbers of A0M and APP and of A0P and AMM state agents), the overall opinion of the whole society is given by the difference between CPP and CMM numbers. This observation may be used to explain the apparent puzzle why in so many highly contested elections the results are close to 50/50 ratio for two major contenders. As already noted, this phenomenon has been addressed by Galam [26] through introduction of the special class of agents called contrarians or by Mobilia [47], who called such inflexible agents zealots. In our case the explanation rests on the observation that the agitated agents are much more likely to participate in the elections. And as their numbers are "automatically" balanced within the model, the results of the election would be close to 50/50 – unless one of the parties excels in its ability of mobilizing the calm agents to vote.

4.3 Subjective perception effects

One of the most interesting effects resulting from the short range of the interactions, is the difference in the global characteristics (given, for example, by the relative ratios of agents of each type) and the corresponding ratios "as seen" by the agents of each type. Here, we define the local perception directly – by the average number of messages of each type, as received by agents in a specific state, for example, the average ratios of messages representing each state as seen by the CMM agents. As it turns out, even when the global dominance of the majority is very strong, the local "view" of a specific agent may be very different. Figure 9 presents the results of such comparison for the simulation in Figure 4. We compare the actual ratios of agents in each of the seven states for the whole population with the averaged values perceived by agents of each type. The "perception" range is taken to be equal to the communication range. Despite the rather long range visibility (the number of communicating agents $N_C = 224$), the local perception of the system composition by certain types of agents may actually be the opposite of the actual social composition. For example the minority calm agents (CPP) communicate mostly within their group, so they see themselves as the majority! The same applies to the APP agents. The small number of neutral agents (C00) see roughly similar number of CPP and CMM agents – which "explains" their neutrality. On the other hand the two large groups of agitated agents (A0P and AMM) perceive approximately correct distribution of agents. We note that the calm majority (CMM) sees the world even more uniformly in their favor than it really is.

5 Conclusions

The results presented in this paper show that the discretized cusp catastrophe model may be a good basis for modeling psychological and social phenomena related to opinion changes. Using only two variables, both of which have direct meaning in the social world and may be measured in psychological experiments or social observations (probability of becoming irritated by calm contrary messages p_a and the range of communication N_C), the model reproduces some important empirical observations. Among these, the most important are:

– difference in perception of the distribution of opinions by specific agents from the true one, especially for the minority (which does not see itself as a minority!);
– absolute (long term) stability of minority views, resulting not from any fixation external to the model dynamics or from cutting of links with the rest of society [55,56], but rather from the individual mechanisms inhibiting the opinion change for the minority agents in the highly emotional state;
– possibility to explain the existence of nearly 50/50 results of many elections, due to the high participation of people in the agitated emotional state, whose numbers are balanced in our model. This phenomenon has been observed in many countries, for example it has been present in the US presidential results since 1988 where the popular vote differences are quite stably below 10%, and the voter turnout is between 50% and 60%. Galam has suggested that the 50/50 effect results from the presence of contrarians [57]. The difference is that in our model, there may exist a calm majority, which may not be motivated enough to participate in the elections. Thus the success in elections may depend on mobilizing the calm supporters into voicing their opinion.

In addition, the model predicts some finite time effects, for example the possible existence of the calm minority communities, surrounded (defended?) by agitated agents. Within the community, all the agents share the same information and opinions. The lifetime of such communities depends on the communication range (ability to "penetrate" the closed community with contrarian information). This dependence is described by a power law, decreasing approximately inversely proportionally to the number of communicating agents. These effects remind of some social situations, but to allow direct comparisons it would be necessary to extend the model by including external influences (propaganda) and using more realistic social network communication topology (e.g., with the presence of highly connected individuals found in the scale free networks). In fact the basic message based communication framework allows a straightforward inclusion of external peopaganda, as already noted in reference [34]. The importance of such influences (due to mass media or cultural trends) has also been studied in different models [58–60]. The basic framework consisting of the nontrivial individual opinion dynamics and the message based communication is meant to be flexible and to allow application to a wide range of environments and to provide grounds for comparison with the empirical studies of actual social systems.

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